Evaluation of roller arrangement of sphere by omnidirectional integral value

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Abstract

Roller arrangement problem is a sphere conveyance problem of driving rollers. In this research, the roller arrangement problem, viewed as an evaluation function, is thought of as mean of roller kinetic energy with respect to the sphere direction. Furthermore, theoretically, we derive the function, and find the contact point such that the evaluated value is minimal.

Keywords: Omnidirectional movement, Angular velocity vector of the sphere, Kinetic energy of the sphere

1. Introduction

A sphere, one of the basic shapes of robots, is used not only as a multi-fingered fingertip mechanism for hand robots but also as an actuator transmission mechanism for omnidirectional movement and drive in mobile robots. Spheres are also used as driving rollers for omnidirectional movement mechanisms, and there are various arrangements, depending on the application of the movement mechanism.

Figure 1 shows the roller contact type for the number of actuators(N_w) per sphere.

Examples of mechanisms driven by two rollers include a power transmission mechanism by Wada et al. [1] (see **Figure 1**(a)), a mobile device using Miyamoto's ball [2] (see **Figure 1**(b)), and. The abovementioned mechanisms can be used for the roller of a wheelchair. In the roller arrangement, a roller fixed in advance is performed on the equator parallel to the horizontal plane, and the sphere can be rotated in two degrees of freedom by generating an angular velocity vector on the plane by Kimura [3].

The ball holding mechanism [4] (see **Figure 1**(c)) is intended to transport the ball, and the roller is placed in the upper hemisphere to hold the ball by friction.

Here, roller arrangement problem is considered a sphere conveyance problem by driving rollers.

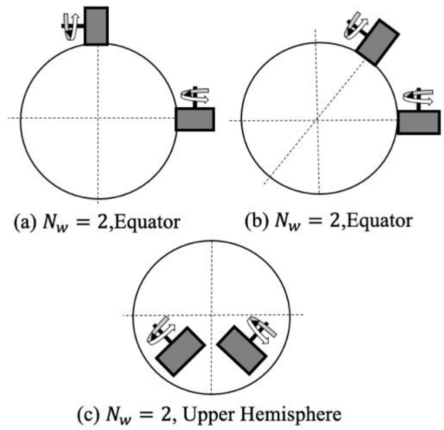

Figure 1 Type of roller arrangement for sphere mobile robot

In this research, in the case of omnidirectional movement, we define an evaluation function as mean of roller's kinetic energy with respect to sphere direction angle, and we also derive the exact formula. Furthermore, theoretically, we find the contact point such that the evaluated value (mean of roller's kinetic energy) is minimal. Additionally, we perform simulation and present energy distribution of several contact points on a sphere.

2. Derivation of theoretical evaluation function

In this chapter, we calculate the omnidirectional energy integral of the driving rollers.

As shown in **Figure 2**, The center $\boldsymbol{O}$ of a sphere with radius r is fixed as the origin of the coordinate system $\Sigma - xyz$. The i^{th} constraint roller (i = 1 or 2) is in point contact with the sphere at a position vector $\boldsymbol{P_i}$ ($\boldsymbol{P_1} \neq \boldsymbol{P_2}$). $\boldsymbol{\omega}$ denotes the angular velocity vector of the sphere. Because of $\boldsymbol{\eta_1}, \boldsymbol{\eta_2} \in \mathbf{span}\{\boldsymbol{P_1}, \boldsymbol{P_2}\}$ (omnidirectional condition), $\boldsymbol{\omega}$ is on span$\{\boldsymbol{P_1}, \boldsymbol{P_2}\}$. sphere direction φ ($0° \leq \varphi < 360°$) is the angle from x-axis and ρ is the angle from xy-plane to $\boldsymbol{\omega}$. Now, given the sphere mobile velocity $\boldsymbol{V}$ (the center velocity of sphere).

2.1 Kinetic energy of the roller

Consider two rollers (right cylinder) with radius R, mass M, moment of inertia I, and roller's angular velocity ω_i . The total kinetic energy of the rollers is given by Eq. (1).

$$E = I(\omega_1^2 + \omega_2^2) \qquad (1)$$
$$= \frac{M}{2}(\|\boldsymbol{\omega} \times \boldsymbol{P}_1\|^2 + \|\boldsymbol{\omega} \times \boldsymbol{P}_2\|^2)$$

2.2 Mean of kinetic energy of rollers

To evaluate the value for roller arrangement, we define the follows expressions. Eq. (2) presents the mean of kinetic energy by integrating the total kinetic energy of the rollers with respect to the direction φ ($0° \leq \varphi \leq 360°$).

$$E_M = \frac{1}{2\pi}\int_0^{2\pi} E \, d\varphi \qquad (2)$$

(i) Case of arbitrary arrangement

Quoting Equation (12) of Paper [5] (Kimura) as follows:

$$\|\boldsymbol{\omega} \times \boldsymbol{P}_1\|^2 + \|\boldsymbol{\omega} \times \boldsymbol{P}_2\|^2 \qquad (3)$$
$$= (\|\boldsymbol{e_3} \times \boldsymbol{P}_1\|^2 + \|\boldsymbol{e_3} \times \boldsymbol{P}_2\|^2)\omega_z^2$$
$$+ 2(\langle \boldsymbol{\acute{\omega}} \times \boldsymbol{P}_1, \boldsymbol{e_3} \times \boldsymbol{P}_1 \rangle + \langle \boldsymbol{\acute{\omega}} \times \boldsymbol{P}_2, \boldsymbol{e_3} \times \boldsymbol{P}_2 \rangle)\omega_z$$
$$+ \|\boldsymbol{\acute{\omega}} \times \boldsymbol{P}_1\|^2 + \|\boldsymbol{\acute{\omega}} \times \boldsymbol{P}_2\|^2$$

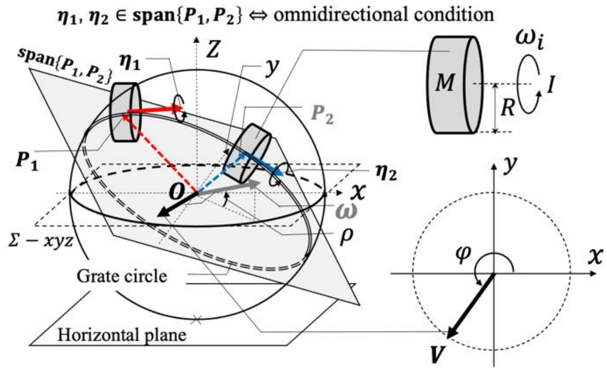

Figure 2 The sphere rotational motion by driving rollers at $\boldsymbol{P_i}$.

where

$$\boldsymbol{P_i} = r[\cos\theta_{i,1} \cos\theta_{i,2}, \sin\theta_{i,1} \cos\theta_{i,2}, \sin\theta_{i,2}]^T \qquad (4)$$

$$\boldsymbol{e_3} = [0, 0, 1]^T, \ \boldsymbol{\acute{\omega}} = [\omega_x, \omega_y, 0]^T, \ \omega_z = \|\boldsymbol{V}\| \tan\rho / r$$

Using $\boldsymbol{P}_i = [P_{i,x}, P_{i,y}, P_{i,z}]^T$, $\boldsymbol{e_3} \times \boldsymbol{P}_i$, $\boldsymbol{\acute{\omega}} \times \boldsymbol{P_1}$ are represented as follow.

$$\boldsymbol{e_3} \times \boldsymbol{P}_i = [-P_{i,y}, P_{i,x}, 0]^T \qquad (5)$$

$$\boldsymbol{\acute{\omega}} \times \boldsymbol{P}_i = [\omega_y P_{i,z}, \omega_x P_{i,z}, \omega_x P_{i,y} - \omega_y P_{i,x}]^T \qquad (6)$$

Using Eqs. (5), $\|\boldsymbol{e_3} \times \boldsymbol{P}_i\|^2$ is calculated in teams of $\boldsymbol{P}_i = [P_{i,x}, P_{i,y}, P_{i,z}]^T$.

$$\|\boldsymbol{e_3} \times \boldsymbol{P}_1\|^2 + \|\boldsymbol{e_3} \times \boldsymbol{P}_2\|^2 \qquad (7)$$
$$= P_{1,x}^2 + P_{1,y}^2 + P_{2,x}^2 + P_{2,y}^2 = 2r^2 - P_{1,z}^2 - P_{2,z}^2$$

Using Eqs. (5) and Eqs. (6),

$$\langle \boldsymbol{\acute{\omega}} \times \boldsymbol{P}_1, \boldsymbol{e_3} \times \boldsymbol{P}_1 \rangle + \langle \boldsymbol{\acute{\omega}} \times \boldsymbol{P}_2, \boldsymbol{e_3} \times \boldsymbol{P}_2 \rangle \qquad (8)$$
$$= -\frac{\|V\|}{r}\{(P_{1,x}P_{1,y} + P_{2,x}P_{2,z}) \sin\varphi$$
$$+(P_{1,y}P_{1,z} + P_{2,y}P_{2,z}) \cos\varphi\}$$

Using Eqs. (6),

$$\|\boldsymbol{\acute{\omega}} \times \boldsymbol{P}_1\|^2 + \|\boldsymbol{\acute{\omega}} \times \boldsymbol{P}_2\|^2 = \frac{\|V\|^2}{r^2}\{P_{1,z}^2 + P_{2,z}^2$$
$$+(P_{1,y}^2 + P_{2,y}^2)\sin^2\varphi + (P_{1,x}^2 + P_{2,x}^2)\cos^2\varphi$$
$$+2(P_{1,x}P_{1,y} + P_{2,x}P_{2,y})\sin\varphi\cos\varphi\} \qquad (9)$$

Thus. By substituting Eqs. (7), Eqs. (8) and Eqs. (9) for Eqs. (3), It can be represented in teams of $P_{i,x}$, $P_{i,y}$, $P_{i,z}$.

Here, using expressions,

$$\frac{1}{2\pi}\int_0^{2\pi}\sin^2\varphi\, d\varphi = \int_0^{2\pi}\cos^2\varphi\, d\varphi = \frac{1}{2}$$

$$\frac{1}{2\pi}\int_0^{2\pi}\sin\varphi\cos\varphi\, d\varphi = 0 \tag{10}$$

Integral of Eqs. (8) and Eqs. (9) by $\varphi(0° \leq \varphi \leq 360°)$ will be calculated as follow.

$$\int_0^{2\pi}(\langle \boldsymbol{\acute{\omega}} \times \boldsymbol{P}_1, \boldsymbol{e_3} \times \boldsymbol{P}_1\rangle + \langle \boldsymbol{\acute{\omega}} \times \boldsymbol{P}_2, \boldsymbol{e_3} \times \boldsymbol{P}_2\rangle)\omega_z d\varphi \tag{11}$$

$$= \pi\frac{\|V\|^2}{r^2}\frac{1}{P_{1,x}P_{2,y}-P_{1,y}P_{2,x}}\{ (P_{1,x}P_{1,z} + P_{2,x}P_{2,z})(P_{1,y}P_{2,z} - P_{2,y}P_{1,z}) + (P_{1,y}P_{1,z} + P_{2,y}P_{2,z})(P_{1,z}P_{2,x} - P_{1,x}P_{2,z})\}$$

$$\int_0^{2\pi}\|\boldsymbol{\acute{\omega}} \times \boldsymbol{P}_1\|^2 + \|\boldsymbol{\acute{\omega}} \times \boldsymbol{P}_2\|^2\, d\varphi \tag{12}$$

$$= \pi\frac{\|V\|^2}{r^2}(2P_{1,z}^2 + 2P_{2,z}^2 + P_{1,x}^2 + P_{1,y}^2 + P_{2,x}^2 + P_{2,y}^2)$$

$$= \pi\frac{\|V\|^2}{r^2}(2r^2 + P_{1,z}^2 + P_{2,z}^2)$$

By substituting Eqs. (3),(7), (11) and (12) into Eq. (1), E_M can be represented as

$$\frac{4r^2}{M\|V\|^2}E_M = \frac{2r^2 - P_{1,z}^2 - P_{2,z}^2}{\left(P_{1,x}P_{2,y} - P_{1,y}P_{2,x}\right)^2}\{\left(P_{1,y}P_{2,z} - P_{2,y}P_{1,z}\right)^2 + \left(P_{1,z}P_{2,x} - P_{1,x}P_{2,z}\right)^2\} + \frac{2}{P_{1,x}P_{2,y} - P_{1,y}P_{2,x}}\{ (P_{1,x}P_{1,z} + P_{2,x}P_{2,z})(P_{1,y}P_{2,z} - P_{2,y}P_{1,z}) + (P_{1,y}P_{1,z} + P_{2,y}P_{2,z})(P_{1,z}P_{2,x} - P_{1,x}P_{2,z})\} + 2r^2 + P_{1,z}^2 + P_{2,z}^2 \tag{13}$$

By theoretical calculation, we get the following properties.

[Property 1]: Optimality of the evaluated value

If $\left(\theta_{1,2}, \theta_{2,2}\right) = (0, 0)$ ($\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ are on the equator), E_M takes the minimal value $M\|V\|^2/2$.

Method of proof is follow. Using inequality

$$(x^2 + y^2)(2r^2 - x^2 - y^2) \geq (px + \alpha y)^2 + (qx + \beta y)^2 \tag{14}$$

, AM-GM inequality and Cauchy-Schwarz inequality, [**Property1**] is proved.

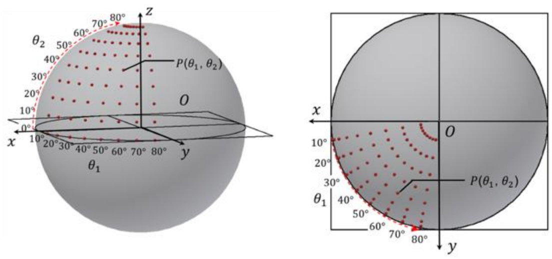

Figure 3 The distribution of contact points on the upper hemisphere. (a) Isometric view. (b) Right overhead view.

Table 1 The distribution of energy function $E_M(\theta_1, \theta_2)$ in the upper hemisphere.

80°	32.19	29.40	25.12	19.87	14.29	9.04	4.76	1.97
70°	8.32	7.67	6.66	5.43	4.12	2.89	1.88	1.23
60°	3.91	3.65	3.25	2.76	2.24	1.75	1.35	1.09
50°	2.38	2.25	2.07	1.83	1.59	1.36	1.17	1.04
40°	1.68	1.62	1.53	1.41	1.29	1.18	1.08	1.02
30°	1.32	1.29	1.25	1.20	1.14	1.08	1.04	1.01
20°	1.13	1.12	1.10	1.08	1.05	1.03	1.02	1.00
10°	1.03	1.03	1.02	1.02	1.01	1.01	1.00	1.00
0°	1	1	1	1	1	1	1	1
θ_2, θ_1	**10°**	**20°**	**30°**	**40°**	**50°**	**60°**	**70°**	**80°**

(ii) Case of symmetry arrangement

Especially, in case of symmetry arrangement ($P_{1,x} = -P_{2,x}, P_{1,y} = P_{2,y}, P_{1,z} = P_{2,z}$), using $(\theta_1, \theta_2) = (\theta_{1,1}, \theta_{1,2})$, $(\theta_{2,1}, \theta_{2,2})$. Eq. (13) is represented as follow.

$$E_M(\theta_1, \theta_2) = \frac{M\|V\|^2}{4r^2}\{2r^2 - 2P_{1,z}^2 + \frac{2P_{1,z}^2(r^2 - P_{1,z}^2)}{P_{1,y}^2}\}$$

$$= \frac{M\|V\|^2}{2}\frac{(1 - \cos^2\theta_1\cos^2\theta_2)}{\sin^2\theta_1} \tag{15}$$

$$(0° < \theta_1 < 90°, 0° \leq \theta_2 < 90°)$$

By theoretical calculation, we prove the following fact.

[Property 2]: Monotonicity of $E_M(\theta_1, \theta_2)$.

(i) When θ_1 increases, $E_M(\theta_1, \theta_2)$ also decrease.

(ii) When θ_2 increases, $E_M(\theta_1, \theta_2)$ also increase.

3. simulation of Evaluation value on sphere

This section presents the simulation results $E_M(\theta_1, \theta_2)$ (Eq.(15)), with $0° < \theta_1 < 90°$, $0° \leq \theta_2 < 90°$, $\|\boldsymbol{V}\| = 1$ [m/s], $M = 2$, and $r = 1$.

Figure 3 shows the contact points on the upper hemisphere. **Table 1** shows the distribution of $E_M(\theta_1, \theta_2)$

at the contact points on the upper hemisphere in steps of θ_1 ($0° < \theta_1 < 90°$) and θ_2 ($0° \leq \theta_2 < 90°$).

As shown in **Table 1**, the value increases from the lower left of the table to the right and upward correspondingly [**Property2**]. $E_M(\theta_1, \theta_2)$ diverges infinitely as it approaches $(\theta_1, \theta_2) = (90°, 0°)$. In particular, when $\theta_2 = 0$, $E_M(\theta_1, \theta_2)$ is constant regardless of the contact position.

As shown in [1] and [2], when two constraint rollers are placed on the equator, the evaluation value is constant regardless of the angle of the two position vectors (see [**Property 1**]).

In the ball holding mechanism (evaluation of the placement of the world team) [4], the roller arrangement is on the upper hemisphere for ball transportation, but it is less-energy efficient than on the equator. Since the ball is not fixed by a pole caster, it is required to be placed on the upper hemisphere.

4. Conclusion

In this research, we defined an evaluation function as mean of roller's kinetic energy with respect to sphere direction angle and derived the exact formula. Furthermore, theoretically, we proved that points on equator are minimal.

Future issues include consideration of motion related to variable mechanisms with offset.

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